Signs of the cusps in binary lenses

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Abstract

The cusps of the caustics of any gravitational lens model can be classified into positive and negative ones. This distinction lies on the parity of the images involved in the creation/destruction of pairs occurring when a source crosses a caustic in a cusp. In this paper, we generalize the former definition of the sign of the cusps. Then we apply it to the binary lens. We demonstrate that the cusps on the axis joining the two lenses are positive while the others are negative. To achieve our objective, we combine catastrophe theory, usually employed in the derivation of the properties of caustics, with perturbative methods, in order to simplify calculations and get readable results. Extensions to multiple lenses are also considered.

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I. Introduction

The application of catastrophe theory in the study of the lens mapping represents a considerable step in the understanding of the critical behaviour in gravitational lensing [?]. Particularly interesting is the classification of singularities through these methods, relying on the evaluation of intrinsic mathematical quantities [?]. A complete treatment of these topics can be found in [?].

The methods of catastrophe theory have proved very powerful in the study of the caustic structure of the binary lens [?] where it has been employed to find the transitions between different topologies, previously studied in the equal mass case [?]. Erdl & Schneider derived the positions of the cusps and showed that, in these transitions, pairs of cusps are created or destroyed passing through beak–to–beak singularities [?]. Witt & Petters [?] used complex notation to study the singularities of the binary lens with an additional shear field and continuously distributed matter.

Since the binary lens is one of the most important lens models, a detailed study of its singularities can help in the interpretation of the physical behaviour of this system and gain information about features that cannot be calculated analytically. For example, the study of the amplification map near the cusps, performed in general by Schneider & Weiss [?], can provide very useful information on the amplification of images in some regimes. Several results on cusp counting in multiple lens systems can be found in [?, ?].

A particular problem is the creation of images in the neighbourhood of cusps. When a source crosses a caustic in a fold singularity, two images of opposite parities are created. The creation of these images happens in a different way when the crossing occurs at a cusp. In this case, one pre–existing image changes parity and two new images of the same parity are created. As the parity of the two new images is the same of the first before the crossing, the sum of the final parities equals the parity of the single image before the crossing.

The parity of the original image involved in this process is a characteristic property of the cusp, called sign [?]. Positive and negative cusps clearly behave in an opposite way, but also the lens mapping in their neighbourhood is influenced in different ways. The sign of a cusp can be determined by a detailed study of the analytical form of the caustic, through the evaluation of the fundamental quantities of catastrophe theory.

In this work, we first give an intrinsic definition of the sign of the cusps and then we use it to determine the signs of the cusps in the caustics of the binary lens. Instead of dealing with the involved exact formulae, we prefer to prove our assertions in some particular cases where perturbative approximations are available [**?**, **?**] and then extend our results by means of continuity arguments.

In Sect. II, we review the principal steps in the description of a cusp by catastrophe theory to state our definition. Sect. III contains the body of the calculation of the signs of the cusps in binary lenses. Sect. IV contains some considerations about the extensions to multiple lenses.

II. Cusps in catastrophe theory

As usual, we introduce the Einstein radius of a reference mass M_0:

$$R_{\mathrm{E}}^0 = \sqrt{4GM_0c^2D_{\mathrm{LS}}D_{\mathrm{OL}}D_{\mathrm{OS}}}. \tag{1}$$

We indicate the coordinates in the lens plane normalized to R_{E}^0 by $\mathbf{x} = (x_1; x_2)$ and the coordinates in the source plane by $\mathbf{y} = (y_1; y_2)$. All masses are measured in terms of M_0. The matter density normalized to the critical density

$$\Sigma_{\mathrm{cr}} = c^2D_{\mathrm{OS}}4\pi GD_{OL}D_{\mathrm{LS}} \tag{2}$$

is $\kappa(\mathbf{x})$.

The Fermat potential of a given distribution of matter is:

$$\phi(\mathbf{x},\mathbf{y}) = 12\left(\mathbf{x}-\mathbf{y}\right)^2 - 1\pi\int \mathrm{d}^2x'\kappa(\mathbf{x}')\ln|\mathbf{x}-\mathbf{x}'|\,. \tag{3}$$

The lens equation is obtained by taking the gradient of this potential:

$$\nabla_{\mathbf{x}}\phi(\mathbf{x},\mathbf{y}) = 0. \tag{4}$$